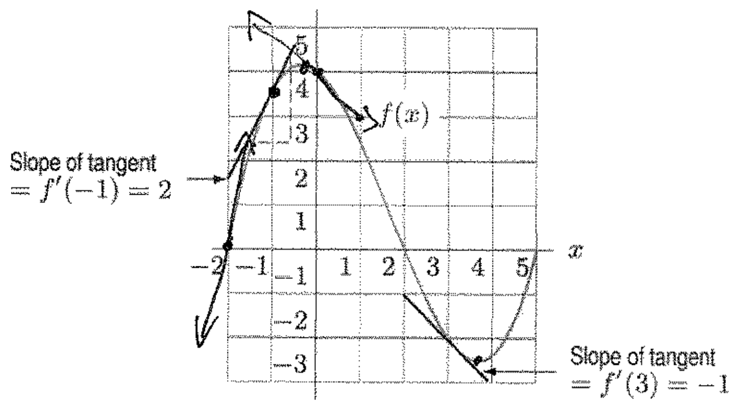
2.2: The Derivative Function

Definition: For a function f, we define the **derivative function**, f', by

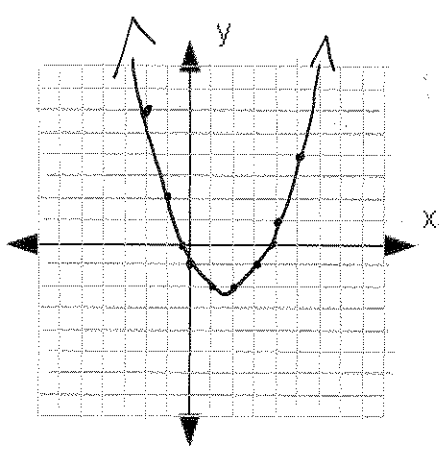
$$f'(x) = \text{ Instantaneous rate of change of } f \text{ at } x = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Example 1: Estimate the derivative of the function $f(x)$ below at $x = -2, -1, 0, 1, 2, 3, 4, 5$.

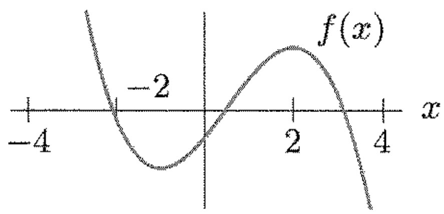


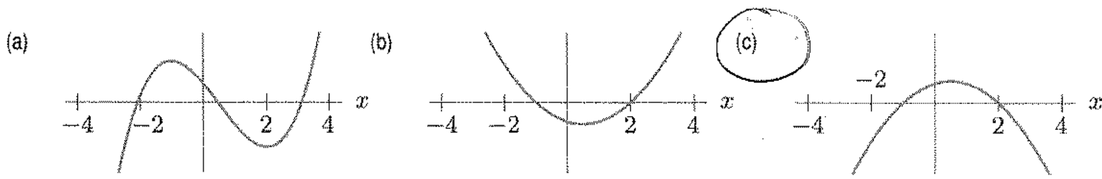
x	-2	-1	0	1	2	3	4	5
Derivative at x	6	2	-1	-2	-2	-1	1	4

Now we can draw the derivative of f.



Example 2: Consider the graph of f below. Which of the graphs (a)-(c) is a graph of the derivative, f'?





The derivative of a graph, f', can tell us a few things about the graph of f itself:

If $f' > 0$ on an interval, then f is *increasing* on that interval.
If $f' < 0$ on an interval, then f is *decreasing* on that interval.
If $f' = 0$ on an interval, then f is *constant* on that interval.

Example 3: A child inflates a balloon, admires it for a while and then lets the air out at a constant rate. If $V(t)$ gives the volume of the balloon at time t, then below is the graph of $V'(t)$ as a function of t. At what time does the child:

(a) Begin to inflate the balloon? $t=3$

(b) Finish inflating the balloon? $t=9$

(c) Begin to let the air out? $t=14$

(d) What would the graph of $V'(t)$ look like if the child had alternated between pinching and releasing the open end of the balloon, instead of letting the air out at a constant rate?